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A foam drainage equation generalized for all liquid contents

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Abstract

Recently the study of liquid drainage through foams has generated considerable experimental and theoretical research. This research has usually made the assumptions that the liquid content is very low and that the viscous resistance is limited to occurring within either the Plateau borders or vertices.

This paper presents a foam drainage model that not only describes the low-liquid-content extreme, but also takes account of liquid content effects. These include the increase in the foam volume with increasing liquid content and the increase in the liquid flowrate in the Plateau borders over the average flowrate brought about by the presence of vertices. The relative importance of the Plateau borders and vertices to viscous loss within the foam is also made dependent on the liquid content of the foam.

This model is verified experimentally for two surfactant types and various bubble sizes using a standard forced drainage system.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Foams play a very important role in a wide range of industrial processes, ranging from the beneficial, such as flotation froths, to the detrimental, such as foaming on distillation column plates. It is for this reason that a fundamental understanding of the processes that occur within froths is required. Important aspects of this understanding are the effects that govern the motion of the liquid within a froth.

Over the last few years there has been much discussion in the foam literature as to the dominant source of viscous losses with a froth. The earliest work on the modelling of foam drainage was carried out by Leonard and Lemlich [1], in which they assumed that the interfaces of the drainage channels were immobile and that the dominant contribution to the losses occurred within the Plateau borders. This work, and the implication of these assumptions, was expanded upon by subsequent researchers [2–4]. The experimental data obtained by these researchers supported the idea that most of the viscous losses occur within the Plateau borders.

More recently, and using a different surfactant system, Koehler *et al* [5] found that the dominant source of viscous losses within their system was the vertices.

The intriguing aspect of these two extremes is that there must exist an intermediate region in which neither of the two sources of viscous loss are dominant. There are two ways in which to move between the extremes; the first is to vary the surface chemistry of the system such that the surface mobility changes, the second is to vary the liquid content in order to change the relative importance of the Plateau borders.

This can be done by carrying out the standard forced drainage experiments as conducted by, for instance, Verbist *et al* [2] and Koehler *et al* [5], but extending the liquid contents examined to higher liquid contents.

2. Mathematical model

2.1. Liquid content

In order to examine theoretically the behaviour of a foam at quite high liquid contents, the assumption that all the liquid is to be found in the Plateau borders [1–4] is no longer valid. Instead the contribution to the volume provided by the vertices must be considered. The contribution to the liquid volume provided by the lamellae will still be ignored.

The liquid content of a foam at each point can be defined by means of two parameters, the radius of the bubbles, r_b , and the radius of curvature of the liquid channels, r . Strictly speaking, knowledge of the polydispersity of the bubble radii is also required, but this effect will be ignored for this case. r is the radius of curvature that would exist if there were only one radius of curvature (such as is the case in a long thin Plateau border) and is more formally defined as follows:

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{P_G - P_L}{\gamma} \quad (1)$$

where r_1 and r_2 are the two radii of curvature of each point, P_G and P_L are the gas and liquid pressure, γ is the surface tension.

The radius of the bubble that is used in this paper is not the radius of the unit cell, defined as the radius of a volume of foam containing one bubble and its associated liquid, but rather the equivalent spherical radius of the gas volume within the bubble. At low liquid contents this subtle difference in definition is not significant, but as the liquid content increases, this difference becomes more important.

A further reason for using the actual bubble radius is that when it is included in the formula for liquid content, the effect of the expansion of the foam with increasing liquid content is inherently accounted for.

It is impossible to define exactly where a Plateau border ends and a vertex begins, since they merge smoothly into one another. However, it is convenient to think of the contributions of these two parts of the channel separately.

This is what was done by Koehler *et al* [5] using data obtained by Phelan *et al* [6] using the surface evolver [7] computer software. Using these data, the liquid fraction, ε , was expressed as a function of L , the typical distance between the centre of two vertices, and r :

$$\varepsilon = 0.171 \left(\frac{r}{L}\right)^2 + 0.2 \left(\frac{r}{L}\right)^3. \quad (2)$$

At the low-liquid-content extreme, this equation can be simplified by the approximation that L is directly proportional to r_b . At high liquid contents this approximation is no longer valid, since for a given bubble radius (r_b , the effective radius of the gas), the distance L will increase with

liquid content as the foam expands. If it is assumed that the unit cells are tetrakaidecahedrons, then L and r_b can be related by means of the following equation:

$$\begin{aligned} \frac{4}{3}\pi r_b^3 &= (1 - \varepsilon) 2^{7/2} L^3 \\ L &= \left(\frac{4\pi}{3 \times 2^{7/2} (1 - \varepsilon)} \right)^{1/3} r_b \\ L &\approx 0.718 \frac{r_b}{(1 - \varepsilon)^{1/3}}. \end{aligned} \quad (3)$$

When equation (3) is substituted into equation (2), a relationship that is implicit in terms of all of its variables is obtained:

$$\varepsilon \approx 0.3316 \left(\frac{r}{r_b} \right)^2 (1 - \varepsilon)^{2/3} + 0.5402 \left(\frac{r}{r_b} \right)^3 (1 - \varepsilon). \quad (4)$$

In terms of the foam drainage equation (24), this formulation of the liquid content does render the finding of analytical solutions very difficult, if not impossible, but the problem can be solved numerically.

2.2. Pressure losses

If it is assumed possible to distinguish between the Plateau border and the vertex, expressions can be written for the pressure loss in each region. We shall theoretically examine a portion of the foam that consists of a single Plateau border and the quarter of the vertex that is to be found at each end.

2.2.1. Plateau border. The Plateau border can be considered to be a tube. The distance between the centre of the two vertices is L , but the length of the Plateau border is shorter than this distance by twice the ‘radius’ of the vertex. The radius of the vertex is proportional to r , with the coefficient of proportionality defined as k_1 . The pressure drop due to viscous losses across this single Plateau border, ΔP_{PB} , can be expressed as follows:

$$\Delta P_{PB} = \frac{C_{PB} v_P \mu (L - k_1 r)}{r^2} \quad (5)$$

where v_P is the average velocity of liquid through the Plateau border, k_1 is the constant relating size of vertex to radius of curvature, μ is the liquid viscosity and C_{PB} is the plateau border loss (‘drag’) coefficient.

For a system in which the interfaces of the Plateau border are rigid, the value of C_{PB} is about 50 [1]. This value will decrease with increasing interface mobility.

2.2.2. Vertex. The flow of liquid in the vertices is far more complex than that within the Plateau borders. Not only does the fluid experience viscous shear brought about by immobile or partially mobile interfaces, it also experiences viscous energy losses due to the bending of the stream lines within the vertex. Even if the interfaces are completely mobile and do not support shearing stresses, they are able to support normal stresses, and thus pressure loss, due to the bending of the fluid. As a first approximation, though, it can be assumed that the pressure losses in the vertex are proportional to the viscosity and velocity of the fluid entering the vertex. Since the only applicable length scale in the vertex is the radius of curvature, r , the pressure drop in the vertex, ΔP_V , can be approximated as follows:

$$\Delta P_V = \frac{C_V v_P \mu}{r} \quad (6)$$

where C_V is the vertex loss (‘drag’) coefficient.

2.3. Losses within a foam

Since there is no clear position at which the vertex ends and the Plateau border begins, there could well be effects that operate at length scales between those expressed above, but they are likely to be far less significant and will be ignored here.

In order to be of practical use, the pressure loss must be expressed in terms of distance within the foam, rather than per Plateau border. Since the Plateau borders are randomly orientated, the average distance between two points along the Plateau borders is three times the straight line distance between the points [2]. The total pressure loss per distance covered in the foam, G , can be expressed as follows:

$$\begin{aligned} G &= \frac{3C_{PB}v_P\mu(L - k_1r)}{Lr^2} + \frac{3C_Vv_P\mu}{Lr} \\ &= v_P\mu \left(\frac{3C_{PB}}{r^2} + \frac{3(C_V - k_1C_{PB})}{Lr} \right). \end{aligned} \quad (7)$$

2.3.1. Low liquid contents. At low liquid contents, certain assumptions can be made:

$$\begin{aligned} L &= 0.718r_b && \text{(from equation (3))} \\ \varepsilon &= 0.171 \left(\frac{r}{L} \right)^2 && \text{(from equation (2))} \\ v_P &= v \end{aligned} \quad (8)$$

where v is the average velocity through the foam, $v = v_s/\varepsilon$, and v_s is the superficial liquid velocity.

The assumptions allow equation (7) to be expressed as

$$G = \frac{v\mu}{r_b^2} \left(\frac{0.995C_{PB}}{\varepsilon} + \frac{2.41(C_V - k_1C_{PB})}{\varepsilon^{1/2}} \right). \quad (9)$$

2.3.2. General case. The general case has a number of added complexities that are not present in the simpler low-liquid-content case. Most of the complexity is a result of the fact that the relationship between r , r_b and ε is no longer trivial. Further, the velocity of the liquid flowing through the Plateau borders, v_P , is not the same as the average velocity, v , since the relative proportion of liquid within the Plateau borders and vertices changes with liquid content:

$$v = \frac{v_s}{\varepsilon} \quad (10)$$

$$v_P = \frac{v_s L^2}{0.171r^2}. \quad (11)$$

This relationship accounts for the cross-sectional area of a typical Plateau border, as well as the number and angle of the Plateau borders in the plane.

Since a formula for ε in terms of r and L is known (2), v_P can be expressed in terms of v , r and L :

$$v_P = v \left(1 + \frac{0.2}{0.171} \frac{r}{L} \right). \quad (12)$$

The portion of the formula following v is the proportion by which the liquid content is higher than it would have been if the Plateau borders joined up without vertices.

The pressure loss per distance in the foam, G , can now be expressed as follows by combining equations (7) and (12):

$$G = v\mu \left(\frac{3C_{PB}}{r^2} + \frac{3 \left(C_V + \left(\frac{0.2}{0.171} - k_1 \right) C_{PB} \right)}{rL} + \frac{3 \frac{0.2}{0.171} (C_V - k_1 C_{PB})}{L^2} \right). \quad (13)$$

By substituting for L using equation (3), the following relationship is obtained:

$$G = v\mu \left(\frac{3C_{PB}}{r^2} + \frac{\frac{3}{0.718} \left(C_V + \left(\frac{0.2}{0.171} - k_1 \right) C_{PB} \right)}{rr_b} (1 - \varepsilon)^{1/3} + \frac{\frac{3}{0.718^2} \frac{0.2}{0.171} (C_V - k_1 C_{PB})}{r_b^2} (1 - \varepsilon)^{2/3} \right). \quad (14)$$

k_1 can be estimated from conditions for close-packed spheres as follows:

$$L = k_1 r_{(\text{Close Packed})} \quad (\text{Plateau borders vanish at close packed limit}) \quad (15)$$

$$r_{(\text{Close Packed})} = 0.5r_b \quad (r_1 \text{ and } r_2 = r_b \text{ at close packed}) \quad (16)$$

$$L \approx 0.794r_b \quad (\text{from equation (3)}) \quad (17)$$

therefore

$$k_1 \approx 1.588. \quad (18)$$

This allows equation (14) to be written as follows:

$$G = v\mu \left(\frac{3C_{PB}}{r^2} + \frac{4.178 (C_V - 0.418C_{PB})}{rr_b} (1 - \varepsilon)^{1/3} + \frac{6.806 (C_V - 1.588C_{PB})}{r_b^2} (1 - \varepsilon)^{2/3} \right). \quad (19)$$

In order to simplify the notation below, the following definition is made so that $G = v\mu k(r, \varepsilon, r_b)$:

$$k(r, \varepsilon, r_b) = \left(\frac{3C_{PB}}{r^2} + \frac{4.178 (C_V - 0.418C_{PB})}{rr_b} (1 - \varepsilon)^{1/3} + \frac{6.806 (C_V - 1.588C_{PB})}{r_b^2} (1 - \varepsilon)^{2/3} \right). \quad (20)$$

It must be remembered that the numerical constants within this formula are a result of the assumption that the bubbles are mono-dispersed and form a Kelvin foam. The values are not likely to vary dramatically as the foam becomes more poly-dispersed, but they will change. The changes are not likely to affect the goodness of fit much, though, seeing that a large amount of any potential error will be absorbed into the two constants C_{PB} and C_V , which can be determined experimentally.

Equation (19) has the same basic length scales as equation (19) of Koehler *et al* [5], though the origin of the third term in the expression is different. In the work of Koehler *et al* [5], the r_b^{-2} term is a result of divergent/convergent flow within the Plateau borders. Within this work, this source of viscous loss is ignored and the r_b^{-2} term is a result of the average velocity of the liquid within the Plateau borders being different from the average velocity through a combination of Plateau borders and vertices. This is why there are only two loss coefficients in equation (19) (C_{PB} and C_V) and three in that of Koehler *et al* [5]. Furthermore, the work of Koehler *et al* [5] is essentially a low-liquid-content description in that it ignores the effect of liquid content on the length of Plateau borders and ignores the contribution of vertices on the overall volume balance.

2.4. Force balance and continuity

The other pressure terms to consider are the gravitational and capillary terms. If it is assumed that the positive velocity is upwards and y is the distance from the foam–liquid interface, then G can be equated to the static head loss and the pressure gradient brought about by changes in the interface curvature. This also assumes that inertial effects are negligible:

$$P_{\text{Liquid}} = P_{\text{Atmosphere}} + \rho g \int_y^{h_f} \varepsilon \, dy - \frac{\gamma}{r} \quad (21)$$

therefore

$$G = -\rho g - \nabla \left(\rho g \int_y^{h_f} \varepsilon \, dy - \frac{\gamma}{r} \right). \quad (22)$$

The average liquid velocity, v , can thus be expressed as follows:

$$v = \frac{-\rho g - \nabla \left(\rho g \int_y^{h_f} \varepsilon \, dy - \frac{\gamma}{r} \right)}{k(r, \varepsilon, r_b) \mu}. \quad (23)$$

The continuity equation that can be used in conjunction with the above equation is the following. This equation is, of course, a simple expression of liquid volume conservation:

$$\frac{\partial \varepsilon}{\partial t} + \nabla(\varepsilon v) = 0. \quad (24)$$

Substituting equation (23) into (24) gives

$$\frac{\partial \varepsilon}{\partial t} - \nabla \left(\frac{\varepsilon \rho g}{k(r, \varepsilon, r_b) \mu} \right) - \nabla \left(\frac{\varepsilon \nabla \left(\rho g \int_y^{h_f} \varepsilon \, dy - \frac{\gamma}{r} \right)}{k(r, \varepsilon, r_b) \mu} \right) = 0. \quad (25)$$

To solve equation (25), the value of r_b is assumed either to be a constant or to be known as a function of position. Either ε or r can be treated as the dependent variable, since they can be determined from one another using equation (4). The only reason why one of them is not eliminated from equations (25) (and (20)) is that equation (4) is implicit in both the variables.

3. Experimental method

Experiments were conducted to obtain empirical data for forced foam drainage. This was achieved by measuring liquid flows and contents within the foam. We thus revisited work of previous researchers [8–10]. Past forced drainage experiments have been conducted using foam column diameters less than 30 mm [5, 8, 11]. The use of such narrow columns means that radial effects can be ignored, significantly simplifying the problem.

3.1. Materials

The surfactants used in the experiments are shown in table 2. Sodium dodecyl 1 sulphate (SDS) was purchased from BDH and tetradecyltrimethyl ammonium bromide (TTAB) was purchased from Fisher Scientific. SDS is a well known foaming agent commonly found in detergents. Its bulk and surface properties have been studied extensively.

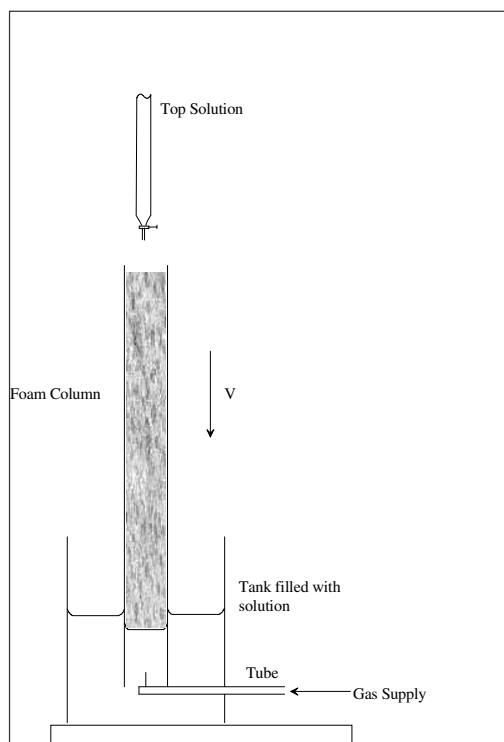


Figure 1. Schematic diagram of experimental apparatus.

Table 1. Variation of parameters to obtain desired bubble sizes.

| | | | |
|---|-------|-------|-------|
| Nominal bubble size (mm) | 1.7 | 2.5 | 5 |
| Orifice diameter | 25 G | 25 G | 1 mm |
| Flowrate, Q_g ($\text{cm}^3 \text{min}^{-1}$) | 16–19 | 70–80 | 70–80 |

3.2. Methodology

A static foam of uniform-sized bubbles was generated in a long vertical column of 14 mm internal diameter. This was achieved by blowing compressed air into the column through a hypodermic syringe needle (see figure 1). Compressed air was supplied at a constant pressure. Since the size of the gas bubbles is dependent on the orifice size and gas flowrate, variation in these parameters gives different bubble sizes. Too high a gas flowrate causes polydisperse bubbles, whereas too low a gas flowrate will create foam of non-uniform density [12]. In order to produce the largest of the bubble sizes investigated (approximately 5 mm) a tube with a 1 mm drilled hole was used instead of the hypodermic syringe needle. The experiments with 5 mm bubbles in the 14 mm diameter column, as would be expected, produced data with substantial wall effects. The experiments with these bubbles were thus conducted in a 34 mm internal diameter column.

The parameters shown in table 1 were used to obtain the desired bubble sizes.

A burette containing the same solution as was used to generate the foam was placed above the foam column, as shown in figure 1. The solution was a mixture of surfactant and distilled water. The concentration of the surfactants was set well above their respective CMCs; the CMCs of the different surfactants was obtained from Mukerjee and Mysels [13] and are shown in table 2.

Table 2. Surfactants and their corresponding CMCs.

| Surfactant | CMC (mM) |
|---|----------|
| Sodium dodecyl 1 sulphate (SDS) | 8.081 |
| Tetradecyltrimethyl ammonium bromide (TTAB) | 3.51 |

As soon as the foam reached a height of 350 mm, the foaming process was stopped and the foam was allowed to dry for approximately 5 min. This ensured the equilibrium state was reached. The next step was to drain solution from the burette into the foam. As soon as solution was added into the foam, a distinctive interface between the wet and dry foam was observed. This front motion was traced down the column and both the flowrate of the solution into the foam (v_s = flowrate of solution/area of column) and the velocity of wave front were noted.

4. Experimental results

One of the useful features of this type of foam drainage experiment is that the shape of the wavefront moving down the column does not change significantly with position [11]. This means that the velocity of the wavefront is the same as the velocity of the liquid upstream of the wavefront. The gas upstream of the wavefront, however, is not stationary. This is because the addition of the liquid is displacing some of the gas. The rate of gas displacement is, of course, the same as the rate of water addition and thus the velocity of the liquid relative to the gas can be written, for this case, as follows:

$$v_{\text{Liquid Rel}} = v_{\text{Wave Front}}(1 + \varepsilon). \quad (26)$$

Since the liquid content in this region is also virtually constant, the effect of interface curvature gradients can be ignored:

$$\nabla \left(\rho g \int_y^{h_f} \varepsilon \, dy - \frac{\gamma}{r} \right)_{\text{Constant } \varepsilon} = -\rho g \varepsilon. \quad (27)$$

The empirically obtained data can be fitted to the following set of equations (from equations (4), (19), (23), (26) and (27)) by adjusting C_{PB} and C_V (it should also be noted that the positive velocities are downwards in these equations):

$$v_{\text{Wave Front}}(1 + \varepsilon) = \frac{\rho g (1 - \varepsilon)}{\mu \left(\frac{3C_{PB}}{r^2} + \frac{4.178(C_V - 0.418C_{PB})}{rr_b} \right) (1 - \varepsilon)^{1/3} + \frac{6.806(C_V - 1.588C_{PB})}{r_b^2} (1 - \varepsilon)^{2/3}}$$

$$v_{\text{Wave Front}} = \frac{v_s}{\varepsilon} \quad (28)$$

$$\varepsilon \approx 0.3316 \left(\frac{r}{r_b} \right)^2 (1 - \varepsilon)^{2/3} + 0.5402 \left(\frac{r}{r_b} \right)^3 (1 - \varepsilon).$$

For each type of surfactant, the parameters C_{PB} and C_V were obtained by means of a best fit to the intermediate-bubble-size (2.5 mm diameter) data. The same values for C_{PB} and C_V were then used in order to predict the performance of the larger and smaller bubble sizes. The assumption being made in this is that the surface viscosity is virtually independent of bubble size, seeing that the deviation of the parameters C_{PB} and C_V from that for rigid interfaces is directly related to the surface viscosity and thus to effects such as surfactant diffusion to and from the interfaces. One of the reasons for using a surfactant solution above the CMC is to eliminate the effect of bulk concentration gradients on the surface viscosity. It can also be seen that the actual sizes of the small, medium and large bubbles are slightly different between the

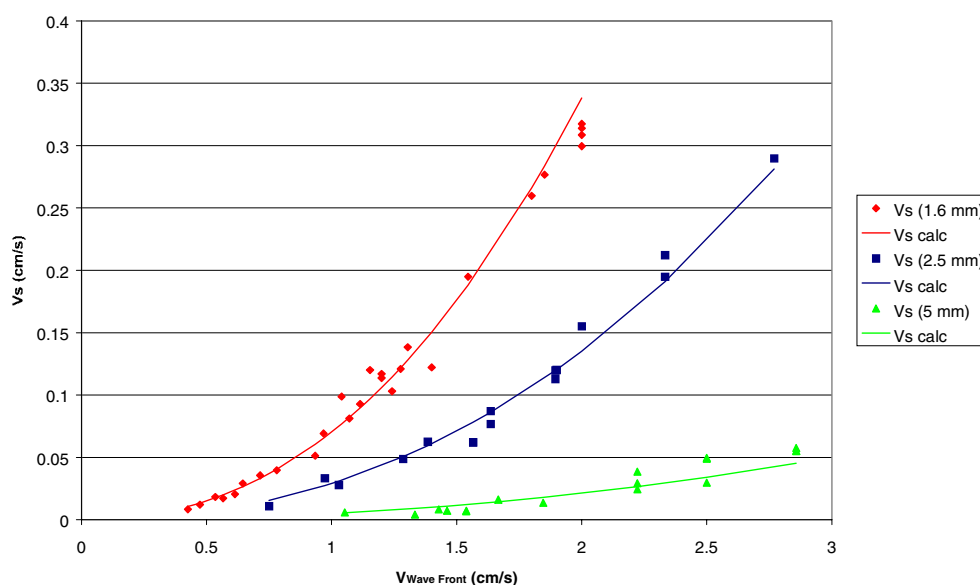


Figure 2. SDS solution: superficial liquid velocity versus interstitial liquid velocity for various bubble sizes ($C_{PB} = 22.3$, $C_V = 23.7$).

two surfactant systems. This is because it is hard to accurately control the size of the bubbles being produced. In each case, though, the bubbles are highly mono-dispersed.

In figure 2, the superficial liquid velocity is plotted against the interstitial liquid velocity for the three different bubble sizes in an SDS solution. It can readily be seen that the theory provides a good fit to the data for the large and small bubbles, even though the two drag coefficients were not obtained using any of these data. The liquid content can also easily be obtained from this graph, seeing that the liquid content is simply the superficial velocity divided by the interstitial velocity. The maximum liquid content seen in these sets of experiments is about 16%, seeing that at not much above this value convective instabilities start to arise within the system.

The values for C_{PB} and C_V obtained for the intermediate bubble size in these SDS experiments are 22.3 and 23.7 respectively. Since we know that the value for C_{PB} when the interfaces are immobile is about 50 [1], the lower value obtained experimentally here indicates a certain degree of interface mobility and thus a finite surface viscosity. It should, in fact, be possible to calculate the surface viscosity using this value and this is the subject of ongoing work.

A very similar set of results was obtained when using TTAB as the surfactant. As before, the drag coefficients were obtained using the intermediate-bubble-size data. The values for the two drag coefficients are 21.9 and 24.5 for C_{PB} and C_V respectively. The difference between the two sets of drag coefficients is very small, which is probably due to TTAB and SDS having quite similar molecular weights (336.4 and 288.3 respectively). The fact that TTAB is cationic and SDS anionic does not seem to influence the results significantly. If a longer-chain molecule were used, it would be expected that the two drag coefficients would increase, with the Plateau border coefficient tending towards a value of about 50.

The slight deviation that can be seen in data for the largest bubbles has two possible causes. Firstly, there could still be some wall effects, even though a larger-diameter column is used in this case. Secondly, the assumption that the surface mobility is the same for all bubble sizes

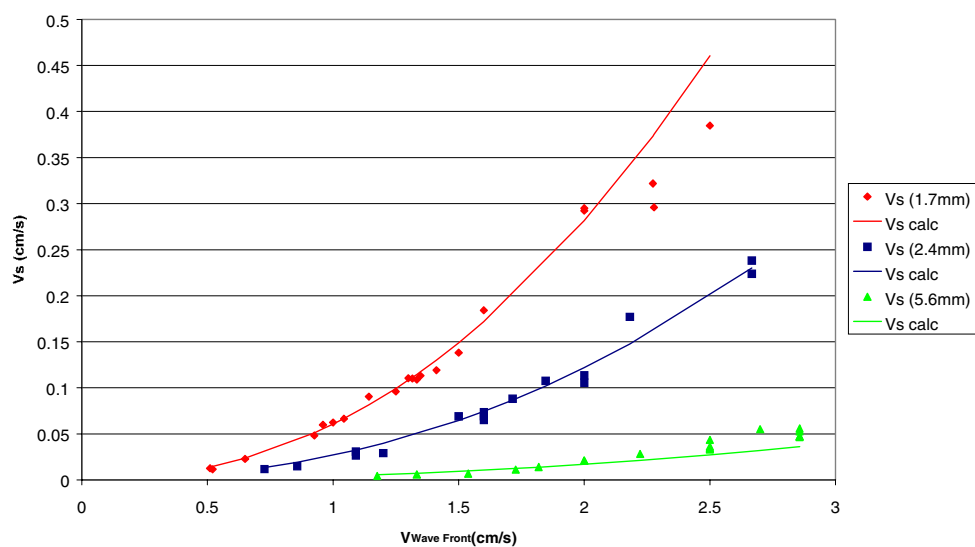


Figure 3. TTAB solution: superficial liquid velocity versus interstitial liquid velocity for various bubble sizes ($C_{PB} = 21.9$, $C_V = 24.5$).

might not be totally appropriate, seeing that a factor in the surface mobility is the ease with which molecules can diffuse to and from the gas liquid interface over the length of the Plateau border. The surface mobility is thus possibly a function of the Plateau border length.

It was noted at the beginning of this paper that there has been a reasonable amount of debate in the literature as to whether the flow through the foam is dominated by the Plateau borders or the vertices. The values for the drag coefficients obtained in this study show that, at least for the surfactant systems studied, neither extreme is wholly appropriate, with the relative importance of the Plateau borders and vertices changing with both bubble size and liquid flow rate.

Figures 4 and 5 show the velocity data for the intermediate bubble sizes, as well as three different best fits to the data. The first of the fits (the bold line) is the same fit as is seen in figures 2 and 3 (the fit to equation (28)). The other two curves on each graph represent the best fits to the Plateau-border-dominated model of Leonard and Lemlich [1] and Verbist *et al* [2] and the vertex-dominated model of Koehler *et al* [5].

It can be seen that for both the TTAB and SDS solutions, the generalized equation provides a better fit to the data. This is, of course, expected, seeing that one of the purposes of this model is to account for the situations that fall between the extremes of Plateau-border- and vertex-dominated drainage. The Plateau-border-dominated fit produced a closer correspondence to the data than the vertex-dominated model. Even though the vertices and the Plateau borders both have similar drag coefficients, at the liquid contents examined here, especially the lower ones, the comparatively larger volume of the Plateau border means that they have the largest effect on the result. As the results of Koehler *et al* [5] indicate, this is not always the case, and if another surfactant system were chosen the effect of the vertices could be much greater.

That the Plateau border model fits the lowest-velocity (liquid-content) data best is not unexpected, seeing that as the liquid content tends to zero, this model and the general model are identical, assuming that the Plateau border drag coefficient is not zero.

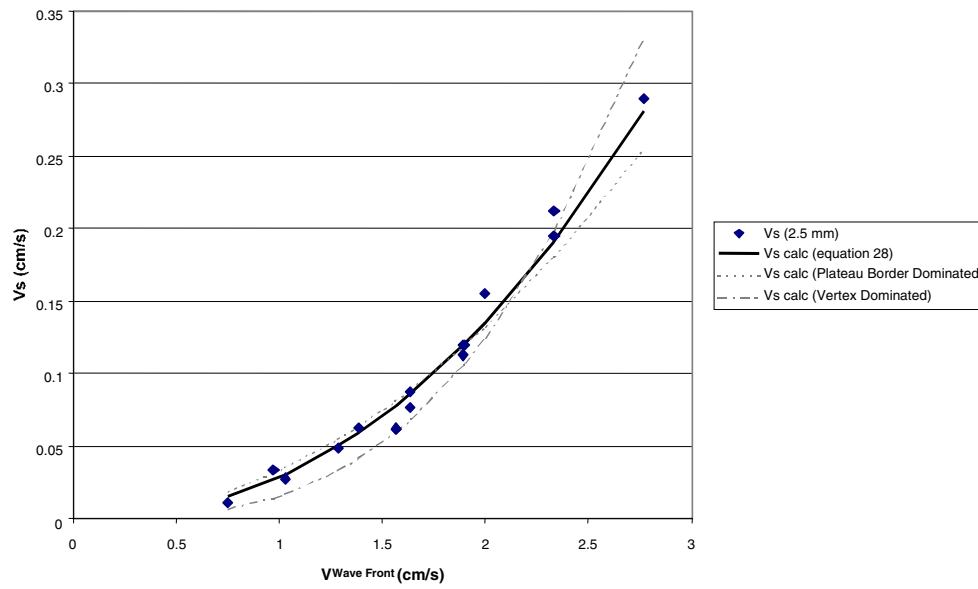


Figure 4. SDS solution. Superficial liquid velocity versus interstitial liquid velocity for the intermediate bubble size: a comparison between the generalized model, the Plateau-border-dominated model and the vertex-dominated model.

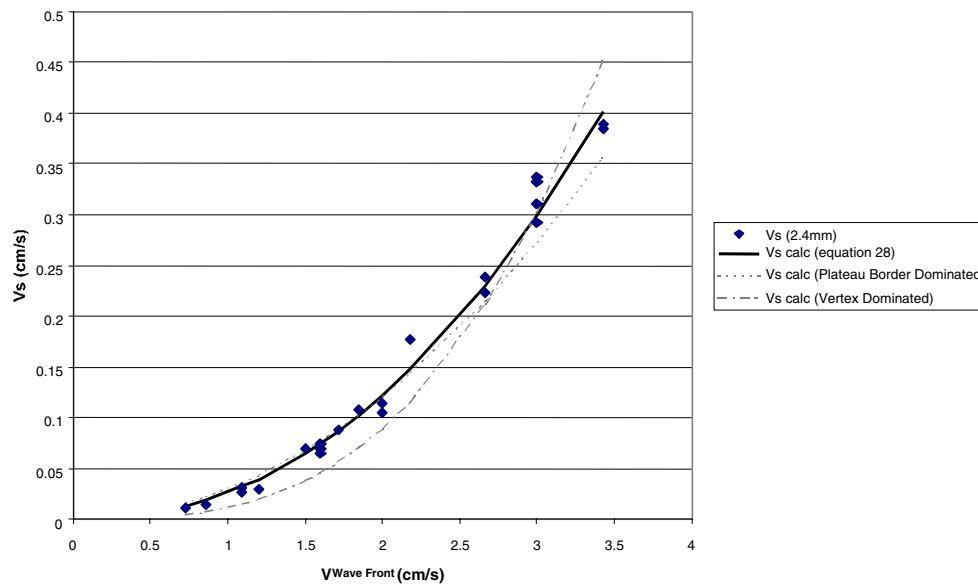


Figure 5. TTAB solution. Superficial liquid velocity versus interstitial liquid velocity for the intermediate bubble size: a comparison between the generalized model, the Plateau-border-dominated model and the vertex-dominated model.

5. Conclusions

An extension of the foam drainage equation has been presented, which can account for the effects of both the vertices and Plateau borders on liquid drainage, as well as removing the

assumption of a low liquid content. The removal of this assumption allows for the effect of foam bed expansion with liquid content to be included. This model also allows for the deviation of the apparent velocity of the liquid when it is in the Plateau borders from the average apparent liquid velocity as liquid content varies.

Despite both vertices and Plateau borders providing a reasonable contribution to the viscous losses in the foam, the model is able to quite accurately fit the experimental data, whereas neither a vertex-dominated, nor a Plateau-border-dominated model is capable of fitting the data over the entire range of liquid contents.

References

- [1] Leonard R and Lemlich R 1965 A study of interstitial liquid flow in foam *A. I. Chem. Eng. J.* **11** 18–25
- [2] Verbist G, Weaire D and Kraynik A M 1996 The foam drainage equation *J. Phys.: Condens. Matter* **8** 3715–31
- [3] Goldfarb I, Kann K B and Shreiber I 1988 Liquid flow in foams *Fluid Dyn.* **23** 244
- [4] Goldschtein V, Goldfarb I and Schreiber I 1996 Drainage waves structure in a gas–liquid foam *Int. J. Multiphase Flow* **22** 991–1003
- [5] Koehler S A, Hilgenfeldt S and Stone H A 2000 A generalised view of foam drainage: experiment and theory *Langmuir* **20** 6327–41
- [6] Phelan R, Weaire D, Peters E A J F and Verbist G 1996 The conductivity of a foam *J. Phys.: Condens. Matter* **8** L475–82
- [7] *Surface Evolver* (University of Minnesota: Geometry Centre) www.geom.umn.edu
- [8] Weaire D, Pittet N, Hutzler S and Pardal D 1993 Steady-state drainage of an aqueous foam *Phys. Rev. Lett.* **71** pp 2670–3
- [9] Koehler S A, Hilgenfeldt S and Stone H A 1999 Liquid flow through aqueous foams: the node-dominated foam drainage equation *Phys. Rev. Lett.* **82** 4232–5
- [10] Durand M, Martinoty G and Langevin D 1999 Liquid flow through aqueous foams: from the Plateau border-dominated regime to the node-dominated regime *Phys. Rev. E* **60** 6307–8
- [11] Weaire D and Hutzler S 1998 Non-linear phenomena in soap froth *Physica A* **257** 264–74
- [12] Weaire D and Hutzler S 1999 *The Physics of Foams* (New York: Oxford University Press)
- [13] Mukerjee P and Mysels K J 1971 *Critical Micelle Concentrations of Aqueous Surfactant Systems* (Springfield, VA: National Technical Information Service)